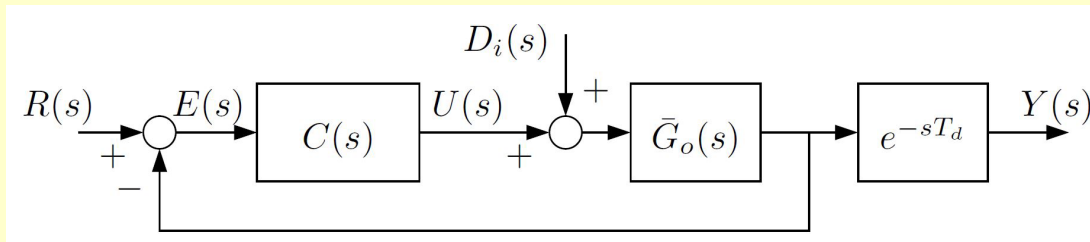


### Introduction

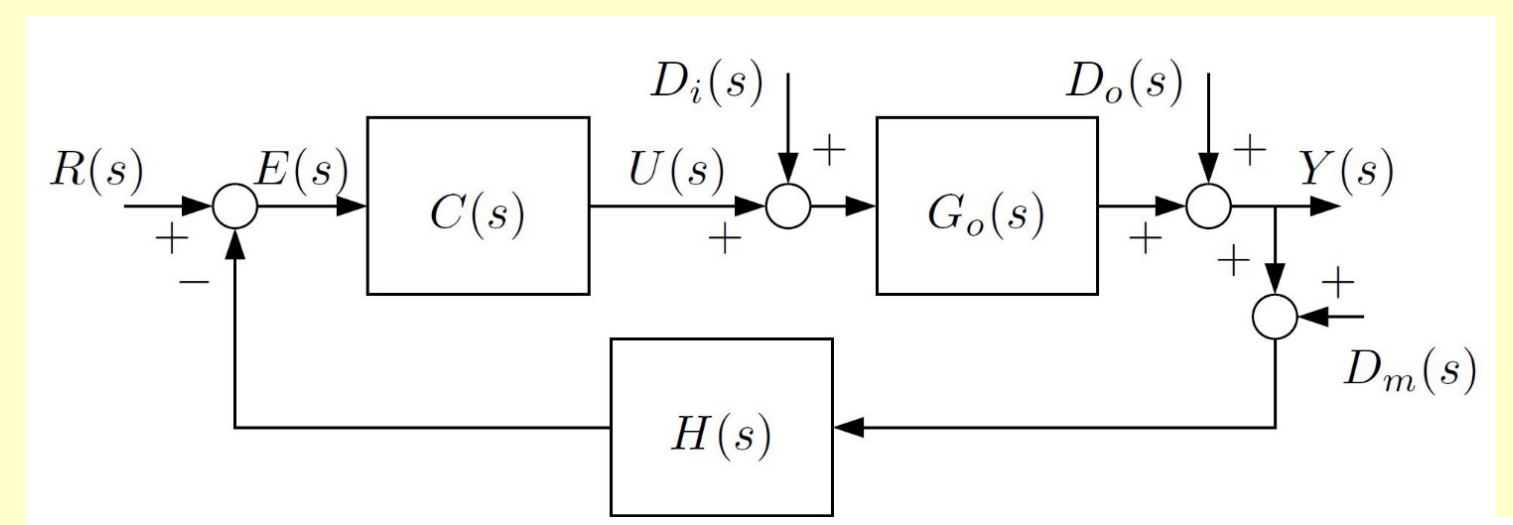
The classical approach when dealing with time-delays is to try to extract the delay from the control loop and design the control for the delay-free plant. In this paper, the effects of the time delay in the controlled plant are analyzed in the frequency domain and filters are designed to counteract these effects.

### Problem Statement

- Given a plant with an I/O time delay  $G_o(s) = e^{-sT_d} \bar{G}_o(s)$
- Introduce a filter  $H(s) = e^{sT_d}$  to "cancel" the delay
- Design the control for  $\bar{G}_o(s)$
- The output will remain delayed.



The perfect filter is not realizable and some approximations are suggested. In this setting, the dynamics of the plant are not so relevant and the approach can be applied for any stable/unstable minimum phase/non-minimum phase plant (or, even, a nonlinear plant). Another option is the use of a notch filter to reshape the frequency response and increase the phase margin of the controlled plant.



### Filter approximations

- Several filter  $H(s)$  approximations are suggested:

$$H_n(s) = \frac{1}{(1 + \tau s)^n} \sum_{\ell=0}^n \frac{1}{\ell!} (sT_d)^\ell \quad n=2 \rightarrow H_2(s) = \frac{1 + sT_d + \frac{1}{2}(sT_d)^2}{(1 + \tau s)^2}$$

and with adjusted high frequency gain

$$H_{2m}(s) = H_2 \frac{(1 + \alpha \tau s)^2}{1 + \alpha sT_d + \frac{1}{2}(\alpha sT_d)^2}$$

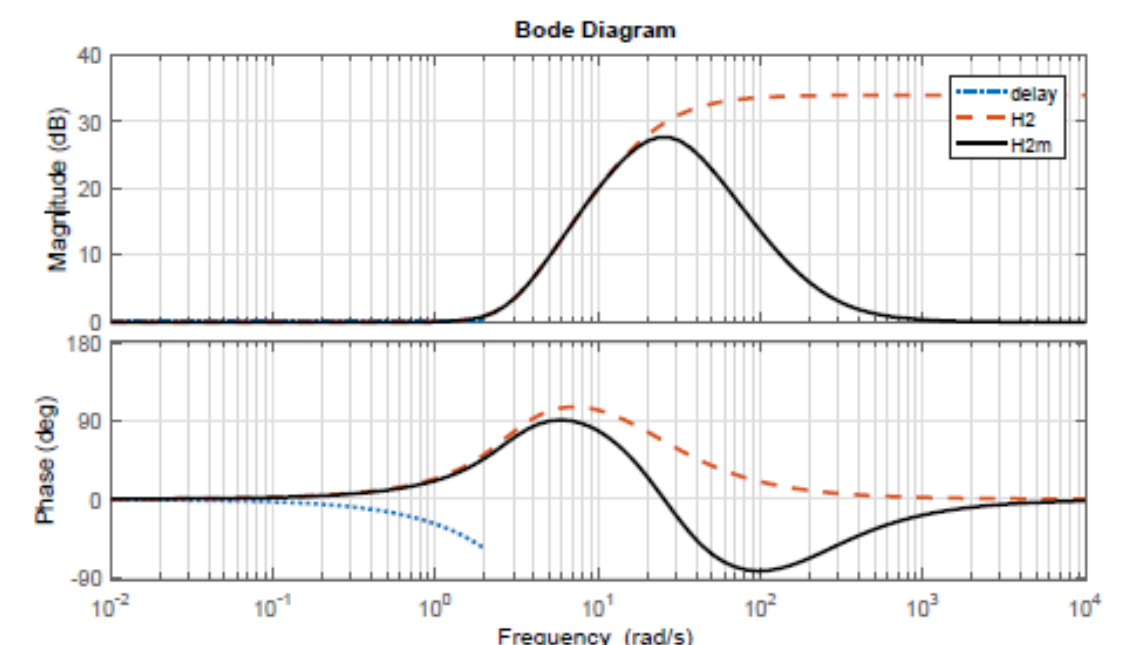


Fig. 5. Bode diagram of  $H_2(s)$  and modified  $H_{m2}(s)$ , cancelling the high frequency gain.

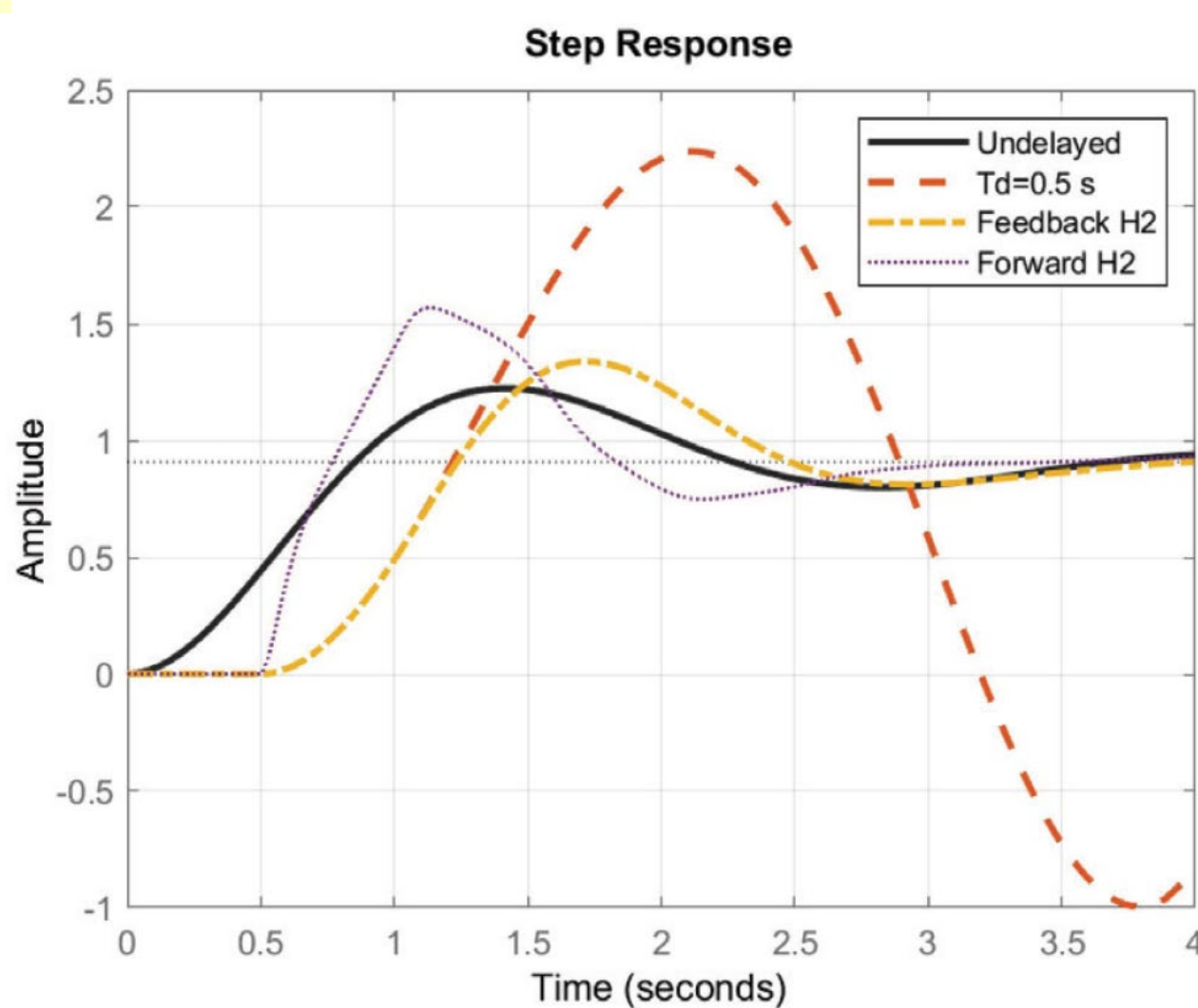
### Control Design: a) Stable plants

$$G(s) = \frac{e^{-T_d s}}{(1 + s)(1 + 2s)} = \bar{G}(s)e^{-T_d s}$$

$$T_d = 0.5 \text{ s}$$

$$H_2(s), \tau = 0.1$$

$$C(s) = 10$$



### Control Design: b) Unstable plants

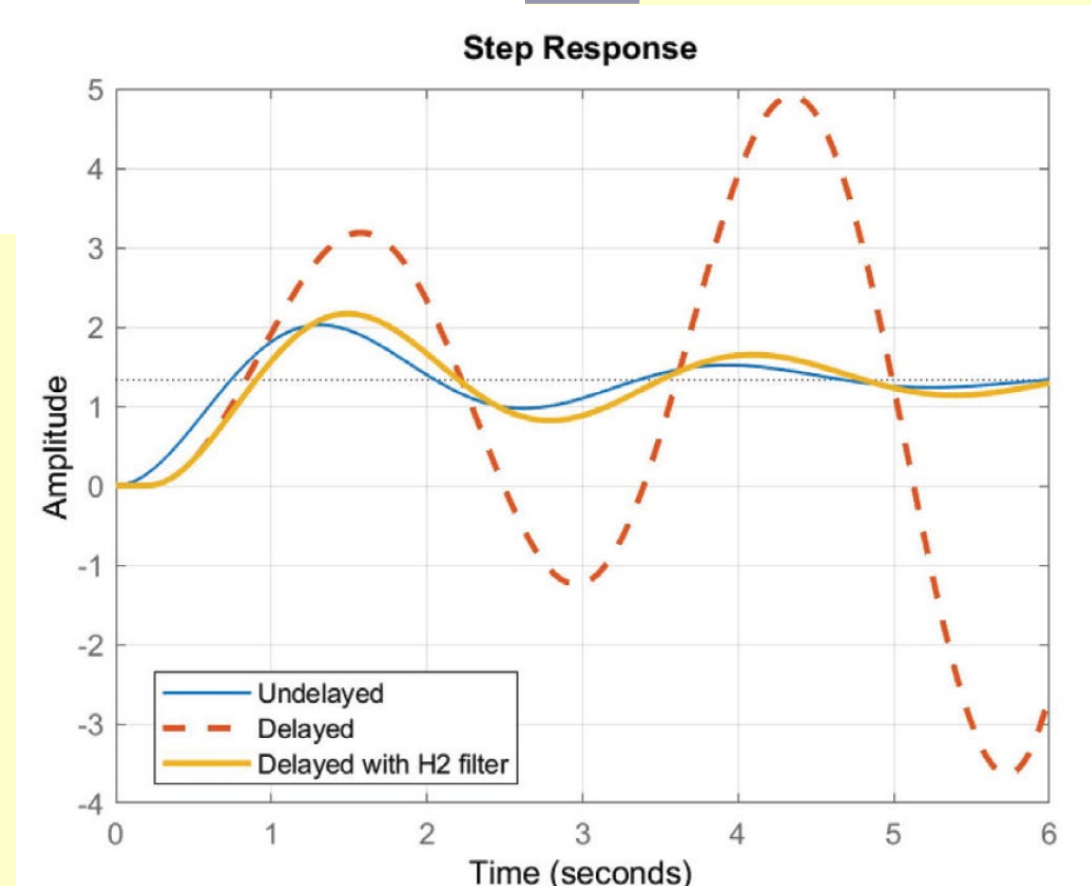
$$G_i(s) = \frac{2}{(s + 2)(s - 1)} e^{-0.2s}$$

$$T_d = 0.2 \text{ s}$$

$$C(s) = 4$$

The small delay makes the system unstable

$$H_2(s), \tau = 0.1$$



### Time delay bound

Looking at the phase at the crossover frequency, the maximum allowable time delay to keep the system stability can be derived:  $T_d < 2/w_c$  as shown in Fig. 9.

### Summary

A time delay approximation in the frequency domain allows an easy time delay compensation for limited range of time delays. A bound for this is derived and the application to design the control for stable and unstable plants has been illustrated by two examples. Other compensation strategies (like the use of Notch filters) have been explored.

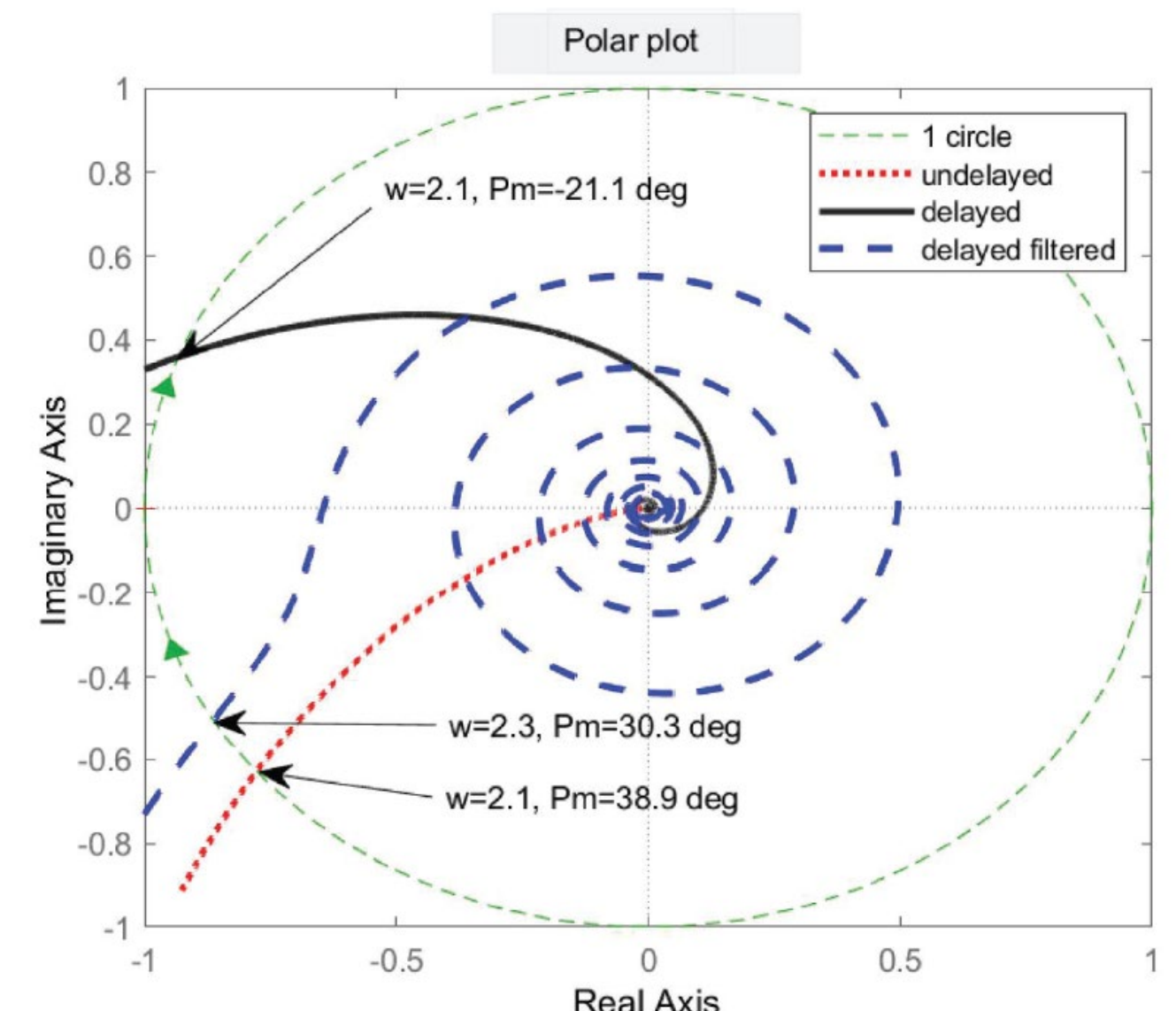


Fig. 9 Polar plot of the loop transfer function: delay-free plant (red dotted line), delayed plant (black solid line) and delayed plant when the filter in the feedback path is  $H_2(s)$  (blue dashed line).